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CORRELATION BETWEEN PARAMETERS AND GEOMETRY IN CASE OF DESIGNING SPIRAL TWIST WITH VARIABLE PITCH

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Abstract:

The aim of the study presented in this paper is to analyze the correlation between the parameters of a spiral twist and the geometry of the tapered and double truncated cone solids on which it is applied, in order to design a helical trajectory with variable pitch, aesthetical harmonized with the shape and sizes of the furniture wood component.

The property of the spiral twist with variable pitch (as helical trajectory on a double truncated surface) is that its axial pitch is proportional to the variable diameter of the wood part on which it is processed, so to create the maximum aesthetic effect of the proportion between the volumes and the dynamism required by the ornament function of the piece of furniture. The diameter size of a double truncated wooden part, which vary along its length, depends both on the position from the edge where the correlation is done and on the piece taper size. The proportion between the pitch measured on the part surface and the corresponding diameter has to comply with the proportion between the axial pitch and the corresponding diameter. The axial pitch cannot be measured, but it can be calculated depending on the pitch measured on the part's surface.

Correlation between the geometric parameters of the spiral twist with variable pitch and the geometry of the designed wooden part on which it is processed (Dmax, Dmin, ∝, Ө) is very important for the aesthetic reason of the ornament and also for defining the kinematics during machining it.

Key words: *spiral twist, variable pitch, longboard, geometry, parameters, tapering*

INTRODUCTION

The ornament and the art of decorative patterning is considered to be a historical characteristic nowadays and has declined in the last century (Strehlke and Loveridge 2005), because of the standardization of components, both in architecture and furniture manufacturing. Researchers deal with the production of digitally generated sculptured ornament using Computer Aided Architectural Manufacturing (CAAD), based on parametric design and 3D modeling of the surfaces (Strehlke and Loveridge 2005, Sun *et al*. 2006, Huertas-Talón *et al*. 2014, Zhou *et al*. 2015).

One of the most complex decoration of the furniture is the spiral-turning leg or column, seen in antique furniture, difficult to be machined, especially on tapered and double truncated cone shafts. This ornament is based on the principle of the helix as used in cutting threads, which is the transposition of the Archimedean spiral on a revolving solid. Its form, size and shape vary according to the aesthetic function of the furniture component. A variation of the spiral may be made in several ways: first, by changing the number of turns of the spiral; second, by running a spiral on a tapered shaft, with variable diameter; third, by changing the shape or form of the spiral itself; and fourth, by making more than one spiral on a shaft (Milton and Wohlers 1919). A fifth way can be and it is the variation of the pitch, which is the distance between two consecutive spirals. The Archimedean spiral has several applications. One of them is the spiral tool path: some papers present a typical mapping from Archimedean spiral to space Archimedean spiral, by generating a new spiral tool path for diamond turning optical freeform surfaces of quasi revolution, close to some surfaces of revolution. (Gong *at al*. 2015) a smooth continuous spiral toolpath plays also an important role for high speed-machining good functioning (Zhou *et al*. 2015), the undesirable mechanizing marks on the machined surface can be avoided by using spiral tool-paths which allow continuous machining of the part without approach or withdrawal (Huertas *et al*. 2014), spiral tool-paths for sculptured surfaces is still being an interesting research aspect, because of the variable surface curvature (Chen and Ye 2002, Sun *et al*. 2006). Other applications of the space Archimedian spiral refers to designing and modeling spiral

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strand cables (Judge *et al*. 2012, Chen *et al*. 2015), to develop quantitative analysis methods in neuroscience research (Miralles *et al*. 2006). Analytical expressions of the cylindrical spiral surfaces with constant pitch were presented by several authors in their research works (Lebedev and Solovjov 2016, Tie *et al*. 2013). Tapered and double truncated cone surfaces are more difficult to be analyzed.

The spiral twist with variable pitch is a helical spiral machined on tapered, double truncated, paraboloid, hyperboloid or spherical surfaces, as presented in Fig. 1. The obtained helix have various profiles, such as semi-circular, oval, pointed arches or triangular ones.

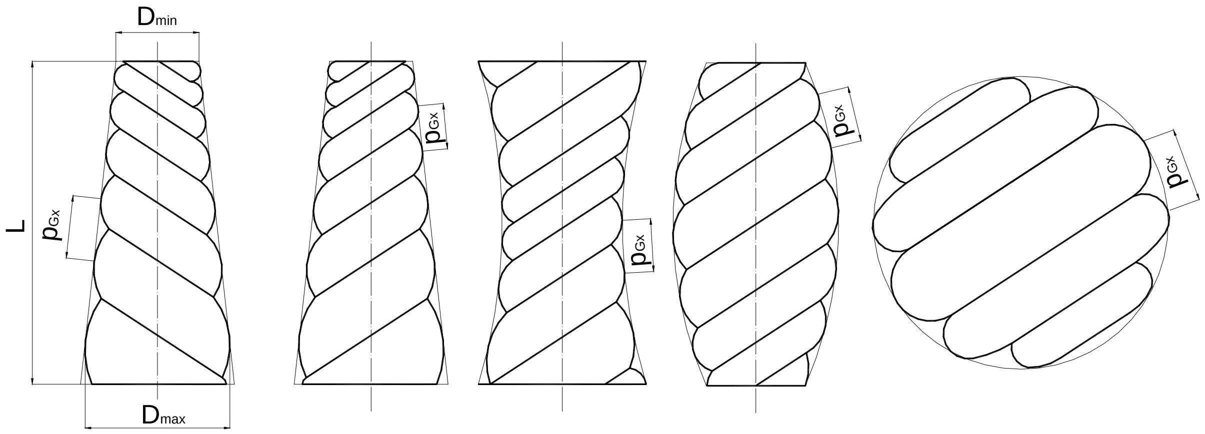

Fig. 1.
Shapes of the furniture elements decorated with spiral twist with variable pitch (L – length of the wooden part; D_{max} and D_{min} – the maximum and minimum diameters of the wooden part; p_{Gx} – variable pitch measured on a certain position of the part's surface.

The pitch of the spiral is the distance between two adjacent helixes and it is measurable at different points and directions, as follows:

- the pitch measured along the trajectory of the surface, namely the generated pitch, p_G;
- the pitch obtained along the axis, namely the axial pitch, p_A;
- the pitch measured perpendicular to the helix, namely the normal pitch, p_N;
- the pitch measured on the crosscut section of the part between two adjacent helixes, namely the frontal pitch p_F.

A geometric correlation must be defined between all the mentioned above parameters and in the same time a correlation between those parameters and the variable diameter D_x has to be found, so that a proportion expressed by an "aesthetic constant rate" to harmonize the ornament to the size and geometry of the furniture element on which it is applied.

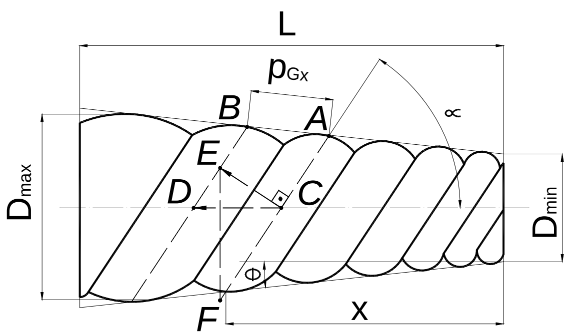

Fig. 2.
Geometric parameters of the spiral twist with variable pitch; D_{min}, D_{max} - diameters at the part ends; ∝ - inclination angle of the ornament; Θ - tapered angle; $\overline{AB}=p_{Gx}$ - pitch measured along the trajectory of the surface at distance x; $\overline{CD}=p_{AX}$ - axial pitch at distance x; $\overline{CE}=p_{Nx}$ - normal pitch at distance x; EF = p_{Fx} – frontal pitch.

The spiral twist with variable pitch, as ornament, is defined by a maximum diameter (D_{max}) and a minimum one (D_{min}) belonging to the wooden element of the furniture, a length L of the element, a taper angle of the element (Θ), and an inclination angle of the helix (α), as presented in Fig. 2. The sizes of the decorated wooden elements are calculated according to the loads applied by the users to the piece of furniture to which they belongs, or as function of proportions between the volume and the weight of the piece of furniture and the decorated element, in the condition of calculating the resistance of the element. In all cases, the tapered wooden element is defined by the geometric elements resulted in Fig. 2.

As presented above, the correlation between the geometric parameters of the spiral twist with variable pitch is more difficult to be done comparing with the case of those decorations with constant pitch (Cismaru and Cosereanu 2014), mainly because of the three-dimensional approach. There are two important reasons to approach the correlation between the parameters and geometry of the spiral twisted wooden furniture elements, and they are as follows:

- to elaborate a design algorithm of this type of ornament, so to obtain an aesthetic dynamism of it and a proportion rate between the volume of the piece of furniture and the sizes of the decorated tapered element;
- to establish a variation rule of the geometric parameters along the wooden element, necessary for the kinematics of processing equipment (Cismaru and Cosereanu 2013).

The correlation parameters-geometry at designing the spiral twist with variable pitch must simplify the process of defining the kinematics and structure of equipment and tools required by processing this type of ornament.

OBJECTIVE

The main objective of the present research is to calculate the parameters of the spiral twist with variable pitch as function of geometric parameters of the wooden part on which the ornament is machined, namely diameters D_{max} and D_{min} of the wooden part and the angles $\propto$ (spiral twist inclination) and Θ (tapered angle of the wooden part). The calculus is made so to obtain a harmonic proportion of the ornament in relation to the furniture part and also for defining the kinematics during its machining.

THEORETICAL APPROACH

The algorithm of the correlation between geometry and parameters of the spiral twisted furniture elements starts with the geometry in a point located at a distance X from one end of the wooden part (Fig. 3).

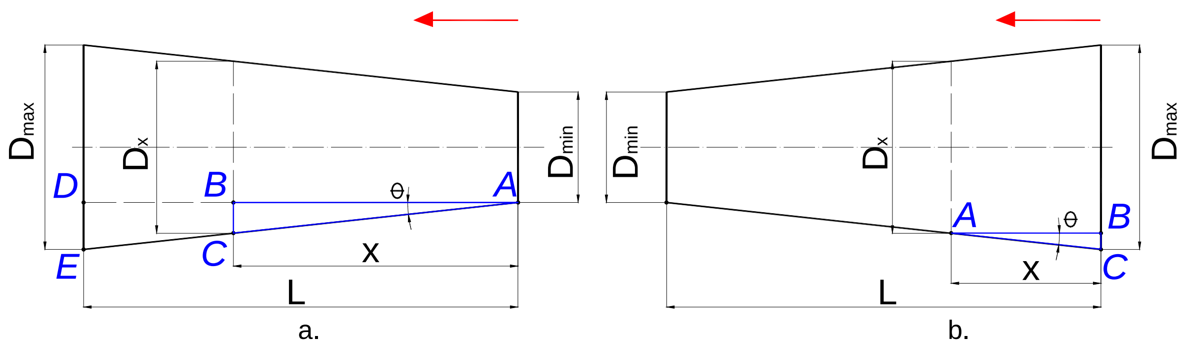

Fig. 3.
Geometry of the wooden elements on which the spiral twist is processed, as basis for the algorithm of the correlation between geometry and parameters of the helix; a – the processing direction starts with D_{min}; b – the processing direction starts with D_{max}.

The mathematical equations applied on the geometric shapes resulted in Fig. 3 are as follows:

➢ The tapered angle Θ resulted from ΔADE in Fig. 3a, where:

$$\frac{\overline{DE}}{\overline{DA}} = tg\theta \quad ; \quad \overline{DE} = \frac{D_{max} - D_{min}}{2} \quad ; \quad \overline{DA} = L$$

and $\theta = arctg\left(\frac{D_{max}-D_{min}}{2L}\right)$ (1)

- The position of a certain point situated at a distance *x* from one end of the wooden part, defined by the diameter D_x:

$$\begin{cases} D_x = D_{min} + 2\overline{BC} = D_{min} + 2x \cdot tg\theta, \text{ for case } \mathbf{a.} \\ D_x = D_{max} - 2\overline{BC} = D_{max} - 2x \cdot tg\theta, \text{ for case } \mathbf{b.} \end{cases} \quad (2)$$

The diameter D_x is defined as a rule of variation of the diameter along the wooden part (Cismaru and Cismaru 2007). In order to obtain a proper aesthetic appearance of the ornament, an accurate and constant proportion between the pitch of the helix and the diameter in any point of the surface must be fulfilled, or mathematically speaking:

$$\frac{p_x}{D_x} = ct.$$

The next step is to choose the right pitch for the proportionality. In this respect, Fig. 4 and a paper published before (Cismaru and Cosereanu 2014) are useful for the decision.

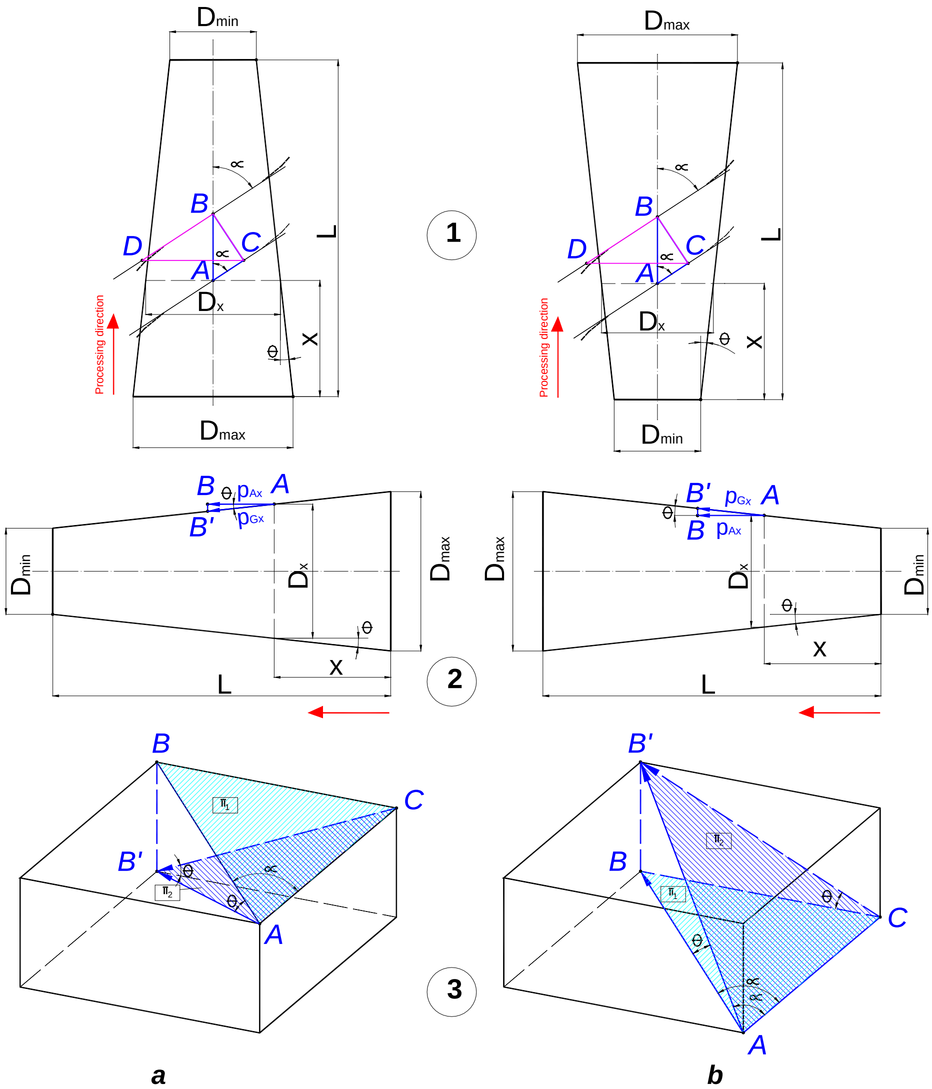

Fig. 4.
Interrelationship between the geometric elements at the spiral twist with variable pitch; a – the processing direction starts with D_{max}; b – the processing direction starts with D_{min}.

The principle of definition and analysis of the spiral twist with variable pitch is the hypothesis that it is composed of an infinite numbers of spiral twists with constant pitches, but having different diameters and overlapped variable lengths. Thus, for each variable point situated at a distance *x* from one end of the wooden part and having the diameter D_x, the correlation between parameters and geometry will be analyzed as for a spiral twist with constant pitch situated on a cylindrical surface having the diameter $D=D_x$. For the situations presented in Fig. 4, the following remarks have to be done:

- $\overline{BC}$ is the normal pitch (p_{Nx})of the spiral twist with constant pitch and diameter $D=D_x$;
- $\overline{AB}$ is the axial pitch (p_{Ax})of the spiral twist with constant pitch and diameter $D=D_x$;
- $\overline{CD}$ is the frontal pitch (p_{Fx})of the spiral twist with constant pitch and diameter $D=D_x$;

In case of spiral twist with constant pitch (Cismaru and Cosereanu, 2014) the following equations were established:

- ➢ $p_N = 2r+a$, where *r* is the radius of the cutting tool and *a* is the distance between two adjacent windings measured at their basis (Fig. 5).

In case of spiral twist with variable pitch (considering $D=D_x$), the above equation can be write as follows:

- ➢ $p_{Nx} = 2r_x+a_x$, or
- ➢ $p_{Nx} = 2r_x+a$, or (3)
- ➢ $p_N = 2r+a_x$.

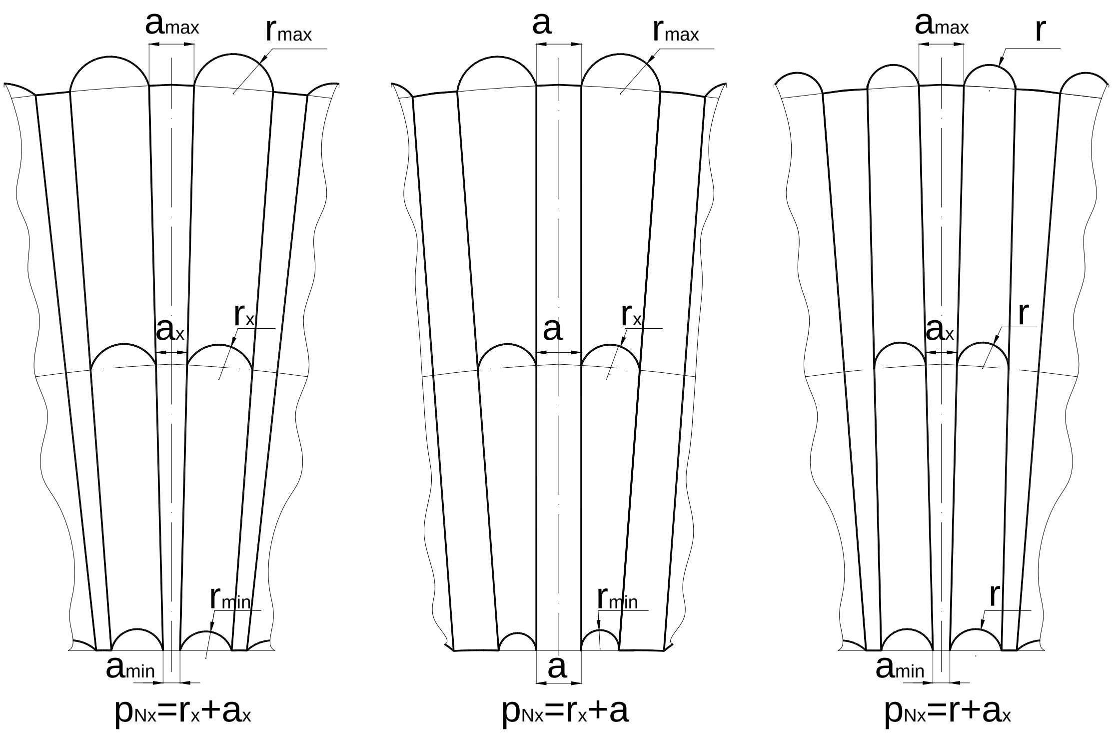

Fig. 5.
Aesthetic variants of obtaining spiral twists with variable pitches.

The three equations of normal pitch (equations 3) give three aesthetic variants of the spiral twist with variable pitch, as shown in Fig. 5, where the unfolded ornament is presented in the three variants. The ornament is supposed to be machined on its length by a tool with constant geometry (r_o=ct. and a_o=ct.). Analyzing the three variants, the following remarks are to be made:

- ➢ the variant presented in Fig. 5a ensures the best proportion and a constant rate between the radius and diameter, as follows:

$$\frac{r_{max}}{D_{max}} = \frac{r_x}{D_x} = \frac{r_{min}}{D_{min}} \quad \text{and} \quad \frac{a_{max}}{D_{max}} = \frac{a_x}{D_x} = \frac{a_{min}}{D_{min}} \tag{4}$$

This variant requires a tool with variable geometry for machining the ornament.

- the variant presented in Fig. 5b ensures only the proportion between the radius and diameter, affecting the aesthetic function of the ornament, requiring also a tool with variable geometry (r_x) for processing the ornament;
- the variant presented in Fig. 5c ensures only the proportion between a_x (distance between two adjacent windings) and the diameter, affecting the aesthetic function of the ornament, but having the advantage of using a tool with constant geometry (*r =ct.*) for ornament processing.

Considering the variant presented in Fig. 5a, even if it requires a tool with variable geometry, the process can be conducted using intersections of the cutting tool trajectories (Cismaru and Cismaru 2007) along the helix, so that the trajectories to be convergent when processing adjacent windings. In this case, equations 4 are transformed as follows:

$$r_x = r_{max}\frac{D_x}{D_{max}} = r_{min}\frac{D_x}{D_{min}} \quad \text{and}$$
$$a_x = a_{max}\frac{D_x}{D_{max}} = a_{min}\frac{D_x}{D_{min}} \qquad (5)$$

In case the equations 2 are introduced in equations 5, equations 5' are obtained.

$$r_x = r_o\frac{D_{max}-2x\cdot tg\theta}{D_{max}} = \frac{r_0}{D_{max}}(D_{max}-2x\cdot tg\theta) \quad \text{for the case in Fig. 4a and}$$
$$r_x = r_{min}\frac{D_{min}+2x\cdot tg\theta}{D_{min}} = \frac{r_{min}}{D_{min}}(D_{min}-2x\cdot tg\theta) \quad \text{for the case in Fig. 4b} \qquad (5')$$

The frontal pitch and the axial pitch can be calculated according to the data shown in the drawings from Fig. 4.

- Frontal pitch (p_{Fx}):

$$p_{Fx} = \frac{\pi\cdot D_x}{Z}$$

where: Z is the number of beginnings, which is a constant of the ornament, resulting from the following equations:

$$Z = \frac{\pi\cdot D_{max}}{p_{Fmax}} = \frac{\pi\cdot D_{min}}{p_{Fmin}} = \frac{\pi\cdot D_x}{p_{Fx}} = ct. \qquad (6)$$

The frontal pitch can be expressed also according to the geometric elements of ΔBCD, as follows:

$$p_{Fx} = \frac{BC}{cos\propto} = \frac{p_{Nx}}{cos\propto} \qquad (7)$$

Using equations 3 and 5, equations 7' are obtained:

$$p_{Fx} = \frac{2r_x + a_x}{cos\propto} = \frac{2r_{max}\cdot\frac{D_x}{D_{max}} + a_{max}\cdot\frac{D_x}{D_{max}}}{cos\propto} = \frac{D_x}{D_{max}\cdot cos\alpha}\cdot(2r_{max}+a_{max}) =$$

$$= \frac{D_{max}-2x\cdot tg\theta}{D_{max}\cdot cos\alpha}\cdot(2r_{max}+a_{max}) \quad \text{for the case in Fig. 4a}$$

where: $r_x = r_{max}\cdot\frac{D_x}{D_{max}}$; $a_x = a_{max}\cdot\frac{D_x}{D_{max}}$; and

$$p_{Fx} = \frac{2r_x + a_x}{cos\propto} = \frac{2r_{min}\cdot\frac{D_x}{D_{min}} + a_{min}\cdot\frac{D_x}{D_{min}}}{cos\propto} = \frac{D_x}{D_{min}\cdot cos\alpha}\cdot(2r_{min}+a_{min}) =$$

$$= \frac{D_{min}-2x\cdot tg\theta}{D_{min}\cdot cos\alpha}\cdot(2r_{min}+a_{min}) \quad \text{for the case in Fig. 4b} \qquad (7')$$

where: $r_x = r_{min} \cdot \frac{D_x}{D_{min}}$; $a_x = a_{min} \cdot \frac{D_x}{D_{min}}$;

The number of beginnings is defined in the maximum crosscut section when designing the ornament, where $r_{max} = r_o$ and $a_{max} = a_o$, and a_{max} can have the following three situations:

- ✓ $a_{max} = a_o$;
- ✓ $a_o < a_{max} \leq 2a_o$; (8)
- ✓ $a_{max} > 2a_o$;

For the case of spiral twist with variable pitch, the value resulted from equation 4 must be also considered, as seen in equation 9:

$$\frac{a_{max}}{D_{max}} = \frac{a_{min}}{D_{min}}; \qquad \text{resulting: } a_{min} = a_{max} \cdot \frac{D_{min}}{D_{max}} \tag{9}$$

A processing condition is imposed in this case, namely the value of a_{min} in three variants:

- ✓ $a_{min} = a_o$;
- ✓ $a_o < a_{min} \leq 2a_o$; (10)
- ✓ $a_{min} > 2a_o$;

Several combinations can occur considering equations 8 and 10 and the aesthetic variants in Fig.5:

✓ $a_{min} = a_o \rightarrow \begin{cases} a_{max} = a_0, \text{adică } a = ct., & \text{not valid} \\ a_0 < a_{max} \leq 2a_0, & \text{valid} \\ a_{max} > 2a_0, & \text{valid} \end{cases}$

✓ $a_o < a_{min} \leq 2a_o \rightarrow \begin{cases} a_0 < a_{max} \leq 2a_0, & \text{valid for the condition } a_{min} \neq a_{max} \\ a_{max} > 2a_0, & \text{valid for the condition } a_{min} \neq a_{max} \end{cases}$

✓ $a_{min} > 2a_o \rightarrow \begin{cases} a_0 < a_{max} \leq 2a_0, & \text{valid for the condition } a_{min} \neq a_{max} \\ a_{max} > 2a_0, & \text{valid for the condition } a_{min} \neq a_{max} \end{cases}$

- Axial pitch (p_{Ax}):

$$p_{Ax} = \overline{AB} = \frac{\overline{BC}}{sin\alpha} = \frac{p_{Nx}}{sin\alpha} \tag{11}$$

If equations 3 and 5 are considered, then equations 11' are obtained.

$$p_{Ax} = \frac{2r_x + a_x}{sin\propto} = \frac{2r_{max} \cdot \frac{D_x}{D_{max}} + a_{max} \cdot \frac{D_x}{D_{max}}}{sin\propto} = \frac{D_{max} - 2x \cdot tg\theta}{D_{max} \cdot sin\,\alpha} \cdot (2r_{max} + a_{max})$$

where: $r_x = r_{max} \cdot \frac{D_x}{D_{max}}$; $a_x = a_{max} \cdot \frac{D_x}{D_{max}}$; and

$$p_{Ax} = \frac{2r_x + a_x}{sin\propto} = \frac{2r_{min} \cdot \frac{D_x}{D_{min}} + a_{min} \cdot \frac{D_x}{D_{min}}}{sin\propto} = \frac{D_{min} + 2x \cdot tg\theta}{D_{min} \cdot sin\,\alpha} \cdot (2r_{min} + a_{min}) \tag{11'}$$

where: $r_x = r_{min} \cdot \frac{D_x}{D_{min}}$; $a_x = a_{min} \cdot \frac{D_x}{D_{min}}$;

Equations 7 and 7' are the rules of variation of both the frontal pitch and axial pitch. These rules are very important for designing the spiral twist with variable pitch and for correlating the parameters with the specific geometry of the wooden part on which the ornament is processed. This rules are also important when processing the ornament with an equipment able to combine the movements of the tool with those of the processed part, so that the trajectories of the adjacent windings to be obtained in the conditions when $r_x \neq$ ct. and $a_x \neq$ ct. along the part on which the ornament is machined.

ALGORITHM OF DESIGN THE SPIRAL TWIST WITH VARIABLE PITCH

The following basic issues are necessary to design the spiral twists with variable pitch using the correlation parameters-geometry:

- a drawing of the wooden part on which the ornament is processed and its geometric parameters: D_{max}, D_{min}, L and Θ;
- a drawing of the tool where r_o and a_o are mentioned.

The geometry of the part and of the ornament together with the data related to the tool are the data base necessary to start the design process according to the following protocol:

- selection of the initial data according to the technical documentation, namely D_{max}, D_{min}, L Θ; r_o and a_o;
- defining the inclination angle of the spiral twist, α, so to give the desired "dynamism" of the ornament;
- establishing the value of the maximum radius of the helix profile in the maximum and minimum crosscut sections, considering the proportionality gives by equation 4:

$r_{max} = r_o$, where r_o is the radius of the tool,

$$r_{min} = r_{max} \cdot \frac{D_{min}}{D_{max}} = r_0 \cdot \frac{D_{min}}{D_{max}}$$

- establishing the distance between the adjacent windings of the helixes in the maximum and minimum crosscut sections, considering the proportionality gives by equation 4:

$a_{min} = a_o$, from the condition of simplifying the work process,

$$a_{max} = a_{min} \cdot \frac{D_{max}}{D_{min}} = a_0 \cdot \frac{D_{max}}{D_{min}}$$

- calculus of the normal pitch in the maximum and minimum crosscut sections:

$$p_{Nmax} = 2r_{max} + a_{max} = 2r_0 + a_0 \frac{D_{max}}{D_{min}};$$

$$p_{Nmin} = 2r_{min} + a_{min} = 2r_0 \frac{D_{min}}{D_{max}} + a_0 \ .$$

- establishing the rule of variation of the normal pitch along the processed part using the following equation of proportion:

$\frac{p_{Nmax}}{D_{max}} = \frac{p_{Nx}}{D_x} = \frac{p_{Nmin}}{D_{min}}$, resulting the following two equations:

$p_{Nx} = p_{Nmax} \cdot \frac{D_x}{D_{max}} = \left(2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}\right) \cdot \left(\frac{D_{max} - 2x \cdot tg\theta}{D_{max}}\right)$, for the case in Fig. 4a;

$p_{Nx} = p_{Nmin} \cdot \frac{D_x}{D_{min}} = \left(2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0\right) \cdot \left(\frac{D_{min} + 2x \cdot tg\theta}{D_{min}}\right)$, for the case in Fig. 4b;

- calculus of the axial pitch in the maximum and minimum crosscut sections:

$$p_{Amax} = \frac{p_{Nmax}}{\sin\propto} = \frac{2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}}{\sin\propto};$$

$$p_{Amin} = \frac{p_{Nmin}}{\sin\propto} = \frac{2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0}{\sin\propto}.$$

o establishing the rule of variation of the axial pitch along the processed part using the following equation of proportion:

$\frac{p_{Amax}}{D_{max}} = \frac{p_{Ax}}{D_x} = \frac{p_{Amin}}{D_{min}}$, resulting the following two equations:

$p_{Ax} = p_{Amax} \cdot \frac{D_x}{D_{max}} = \left(\frac{2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}}{sin\propto}\right) \cdot \left(\frac{D_{max} - 2x \cdot tg\theta}{D_{max}}\right)$, for the case in Fig. 4a;

$p_{Ax} = p_{Nmin} \cdot \frac{D_x}{D_{min}} = \left(\frac{2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0}{sin\propto}\right) \cdot \left(\frac{D_{min} + 2x \cdot tg\theta}{D_{min}}\right)$, for the case in Fig. 4b;

o calculus of the frontal pitch in the maximum and minimum crosscut sections:

$p_{Fmax} = \frac{p_{Nmax}}{cos\propto} = \frac{2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}}{cos\propto}$;

$p_{Fmin} = \frac{p_{Nmin}}{cos\propto} = \frac{2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0}{cos\propto}$.

o establishing the rule of variation of the frontal pitch along the processed part using the following equation of proportion:

$\frac{p_{Fmax}}{D_{max}} = \frac{p_{Fx}}{D_x} = \frac{p_{Fmin}}{D_{min}}$, resulting the following two equations:

$p_{Fx} = p_{Fmax} \cdot \frac{D_x}{D_{max}} = \left(\frac{2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}}{\cos\propto}\right) \cdot \left(\frac{D_{max} - 2x \cdot tg\theta}{D_{max}}\right)$, for the case in Fig. 4a;

$p_{Fx} = p_{Fmin} \cdot \frac{D_x}{D_{min}} = \left(\frac{2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0}{cos\propto}\right) \cdot \left(\frac{D_{min} + 2x \cdot tg\theta}{D_{min}}\right)$, for the case in Fig. 4b;

o calculus of the number of beginnings (parallel helixes):

$Z_e = \frac{\pi \cdot D}{p_F} = ct.$ the number of beginnings is a constant along the entire processed part,

$$\begin{cases} Z_e = \frac{\pi \cdot D_{max}}{p_{Fmax}} = \frac{\pi \cdot D_{max} \cdot \cos\alpha}{2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}}, \text{for the case in Fig. 4a,} \\ Z_e = \frac{\pi \cdot D_{min}}{p_{Fmin}} = \frac{\pi \cdot D_{min} \cdot \cos\alpha}{2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0}, \text{for the case in Fig. 4b.} \end{cases}$$

o rounding the value of number of beginning to an integer number:

$Z_e = Z_i$

o correlation between the parameters and geometry of the spiral twist with variable pitch processed on a tapered wooden part, considering the integer number of beginnings. Several variants are obtained, as follows:

- changing the helix inclination angle, the other parameters remaining constant at the initial values:

$\propto_R = arccos\left[\frac{Z_i \cdot \left(2r_0 + a_0 \cdot \frac{D_{max}}{D_{min}}\right)}{\pi \cdot D_{min}}\right]$, for the case in Fig. 4a,

$\propto_R = arccos\left[\frac{Z_i \cdot \left(2r_0 \cdot \frac{D_{min}}{D_{max}} + a_0\right)}{\pi \cdot D_{min}}\right]$, for the case in Fig. 4b,

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- modifying the normal pitch and of the distance amax between the adjacent windings, the other parameters remaining constant at the initial values:

$p_{F_R} = \frac{\pi \cdot D_{max}}{Z_i}$, which is the frontal pitch correlated with the integer number Z_i in the maximum crosscut section,

$$p_{N_R} = p_{F_R} \cdot \cos\alpha = \frac{\pi \cdot D_{max}}{Z_i} \cdot \cos\alpha,$$

$$p_{N_R} = 2r_0 + a_{max_R},$$

$$\frac{\pi \cdot D_{max}}{Z_i} \cdot \cos\alpha = 2r_0 + a_{max_R}, \Rightarrow a_{max_R} = \frac{\pi \cdot D_{max}}{Z_i} \cdot \cos\alpha - 2r_0$$

- modifying the maximum and minimum diameters, maintaining the tapering angle, the other parameters remaining constant at the initial values:

$p_{N_R} = \frac{\pi \cdot D_{max_R}}{Z_i} \cdot \cos\alpha = 2r_0 + a_{max}$, in the maximum crosscut section resulting:

$$D_{max_R} = \frac{Z_i \cdot (2r_0 + a_{max})}{\pi \cdot \cos\alpha}$$

$D_{max_R} = D_{min_R} + 2L \cdot tg\,\theta$, from Fig. 3, resulting the following equation:

$$D_{min_R} = D_{max_R} - 2L \cdot tg\,\theta = \frac{Z_i \cdot (2r_0 + a_{max})}{\pi \cdot \cos\alpha} - 2L \cdot tg\,\theta.$$

RESULTS AND DISCUSSIONS

When correlating the geometric parameters using the integer value of the number of the beginnings in the algorithm of design, the following results are obtained:

- the simplest solution is to modify the helix inclination angle (α), without changing the “dynamism” of the ornament. A drawing with the new inclination angle (α_R) has to be done and analyzed. If the result is satisfactory, this new correlation is maintained;
- a new correlation by changing the normal pitch and the distance between adjacent windings (a_{max}) is not very complex, but has influence upon the aesthetic function of the ornament. The analysis has to be done on a new drawing with modified a_{max_R} parameter and with the condition $a_{min}=a_{o;}$
- a new correlation of D_{max_R} and D_{min_R} is also simple and fast, with the condition of maintaining the tapering angle, ϴ.

The final decision of correlation between parameters and geometry of the spiral twist with variable pitch processed on tapered wooden surfaces may be taken only after analyzing the calculus results and the graphic design, so to ensure a satisfactory aesthetic function of the ornament.

CONCLUSIONS

Design of spiral twist with variable pitch processed on tapered wooden surfaces using the correlation between the helix parameters and the geometry of the tapered solids imposes an analytical and graphic methodology. The geometric elements are defined using the algorithm of design presented above, then verified through the graphic design of the ornament, with the aim of fulfilling both the aesthetic and the resistance functions of the decorated wooden part integrated in a piece of furniture.

Using the proportion between the geometric elements of the ornament and of the decorated component, the dimensional correlations will help to control the ornament design, so that a satisfactory aesthetic function to be obtained.

The basic parameters and geometric elements selected at the beginning of using the algorithm of design this type of ornament are D_{max}, D_{min}, L, Θ; r_o and a_o. That means that the furniture component is dimensioned so, that both the strength and the proportionality with the piece of furniture where it belongs to be fulfilled. Additionally, the designer can use the algorithm presented in the paper in order to control the “dynamism” of the ornament and its aesthetic function, introducing parameters like α, r_{max}, r_{min}, a_{max} *and* a_{min}. Using the graphic design, the designer can correlate the results of the calculus with the obtained dimensions. The algorithm presented in the paper helps also at defining the rules of variation of several significant parameters, such as p_{Nx}, p_{Ax}, p_{Fx} *and* p_{Gx}, helping thus the industrial processing of this complex ornament and the basics in technological adjustments.

The design algorithm of the spiral twist with variable pitch helps in the final correlation of the parameters, after calculating the number of the helix beginnings and rounded them at an integer value. In this case the correlation between the geometric elements has to be completed by a graphic analysis of the ornament. Thus, the algorithm can be applied easily and effectively, offering flexibility in designing the complex ornament as the spiral twist with variable pitch is.

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